

New Physics in $e^+e^- \rightarrow Z\gamma$ at the ILC with polarized beams: Explorations beyond conventional anomalous triple gauge boson couplings

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ABSTRACT: One of the most-studied signals for physics beyond the standard model in the production of gauge bosons in electron-positron collisions is that due to the anomalous triple gauge boson couplings in the $Z\gamma$ final state. In this work, we study the implications of this at the ILC with polarized beams for signals that go beyond traditional anomalous triple neutral gauge boson couplings. Here we report a dimension-8 CP-conserving $Z\gamma Z$ vertex that has not found mention in the literature. We carry out a systematic study of the anomalous couplings in general terms and arrive at a classification. We then obtain linear-order distributions with and without CP violation. Furthermore, we place the study in the context of general BSM interactions represented by $e^+e^-Z\gamma$ contact interactions. We set up a correspondence between the triple gauge boson couplings and the four-point contact interactions. We also present sensitivities on these anomalous couplings, which will be achievable at the ILC with realistic polarization and luminosity.

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1 Introduction

The Standard Model (SM) is a well-established theory now and is being tested at very high precision in a variety of sectors, e.g., in the Higgs sector at the Large Hadron Collider (LHC), and in the flavour sector at low-energy and high-intensity experiments, to name a couple of examples. Furthermore, the gauge sector of the SM is predictive and highly constrained. The study of gauge-boson pair production will be an important process to look for new physics at the International Linear Collider (ILC) [1, 2]. The ILC is a proposed next generation collider after the LHC that will collide electrons and positrons at high energy and luminosity. The availability of beam polarization, either longitudinal or transverse, of one or both of the beams, will also significantly enhance the sensitivity to new physics interactions [3, 4]. The rate for gauge-boson pair production will be sensitive to the gauge-boson self-interactions, which arise through the non-Abelian nature of the electroweak sector $SU(2)_L \times U(1)_Y$. Thus, it would be important to look for deviations from SM predictions in this sector. Nevertheless, gauge invariance and Lorentz invariance as well as renormalizability place powerful constraints on the possible structures that can arise. Thus a model independent classification of terms has been a rich and highly developed field, see refs. [5–10]. The work of Hagiwara et al. [5] will be used by us as a standard

touchstone in the considerations associated with anomalous couplings in the neutral-boson sector.

Of the many diboson processes that have been considered, $e^+e^- \rightarrow Z\gamma$ has received substantial attention in the past. The $Z\gamma Z$ and $Z\gamma\gamma$ couplings are absent at tree level, and also highly suppressed when allowed by internal particle loops in the SM, forbidding the s -channel production of ZZ and $Z\gamma$. Therefore any deviation from the tree-level SM predictions will signal the presence of beyond-SM (BSM) physics. We will first return to the anomalous couplings for this process that were introduced some decades ago [5, 8, 10]. In particular, these authors have provided a standard basis, in terms of 8 couplings, denoted by h_i^V , $V = Z, \gamma$, $i = 1, 2, 3, 4$, with $i = 1, 2$ denoting dimension-6 and -8 CP-violating couplings while $i = 3, 4$ denote dimension-6 and -8 CP-conserving couplings. The individual values of these triple gauge boson couplings (TGCs) as described before are zero at tree level in the SM, with non-zero values arising at higher orders or in composite models. These anomalous couplings have been extensively studied in the literature in the context of different colliders¹ [11–19]. Moreover there has been a lot of work in the literature [8, 10, 13, 16, 18] where effective Lagrangians or effective momentum-space vertices and the associated form factors in neutral gauge boson production have been discussed.

In all previous work on the subject, there have been no deviations from the set initially considered by Ref. [5], in which the terms are implicitly symmetric under the interchange of $Z \leftrightarrow \gamma$. The lowest-dimension effective operators within the effective Lagrangian approach for the neutral anomalous couplings, with all the particles being off-shell, has been discussed in Ref. [20, 21]. In that work, there is the possibility that there can be terms that do not respect this symmetry at the Lagrangian level. However, we have checked that even those terms produce the same anomalous TGCs. In the present work, we have tried to push this hypothesis further, and indeed at dimension-8 we uncover a new term. Here, we report our finding that an additional coupling involving only the Z exists, with $Z\gamma Z$ coupling consistent with Lorentz invariance, electromagnetic gauge invariance and Bose symmetry, which has not been explicitly reported in the literature. We consider this to be an important addition to the body of literature on anomalous TGCs.

Searches for these neutral anomalous couplings have been performed at LEP [22, 23], the Tevatron [24, 25] and the LHC. The most stringent bounds have come from the ATLAS [26] and CMS [27] collaborations, with the data taken at $\sqrt{s} = 7$ TeV.

¹While the issue of anomalous triple gauge bosons has been discussed for several decades now, there have been inequivalent definitions in the literature. For instance, in Ref. [11] it is mentioned that they have a parametrization which is similar to, but not exactly the same as that of Hagiwara et al. [5]. The form factors of the two are related by an overall normalization, with the form factors of Ref. [5] being (-2) times those of Ref. [11]. In Ref. [12] the effective CP-violating Lagrangian has been written down, and the anomalous couplings are denoted by λ_1 and λ_2 . In our work [17], we have demonstrated that these are equivalent to f_1 and f_2 of Ref. [11].

Since the anomalous gauge couplings would give rise to photons with large transverse momentum, p_T^γ , the LHC collaborations have placed limits on the couplings by measuring the total production cross section and looking at the p_T distribution of the photon. As the photon transverse energy spectrum has similar sensitivity to CP-conserving and CP-violating couplings, the experimental results are generally in terms of the CP-conserving couplings h_3^V and h_4^V . These analyses are all based on what are claimed to be the most general Lorentz invariant effective interactions given by Ref. [5]². The limits are as follows :

- ATLAS : $|h_3^Z| < 0.028$, $|h_4^Z| < 1.74 \times 10^{-4}$, $|h_3^\gamma| < 0.027$, $|h_4^\gamma| < 1.58 \times 10^{-4}$ [26]
- CMS : $|h_3^Z| < 5.4 \times 10^{-3}$, $|h_4^Z| < 2.6 \times 10^{-5}$, $|h_3^\gamma| < 4.9 \times 10^{-3}$, $|h_4^\gamma| < 2.5 \times 10^{-5}$ [27]

It has been pointed out by the authors of [28–31] that one economical way of fingerprinting BSM physics is to use model-independent contact interactions. In the present work, we approach the question of studying the distributions produced by the anomalous couplings in relation to those produced by contact terms, as there has been no detailed comparison of these approaches. We have tried, in as general a manner as possible, to rewrite the anomalous TGC occurring in $e^+e^- \rightarrow Z\gamma$ in terms of contact-type interactions. As it happens, the effective couplings from the former (anomalous TGC) after reducing to effective couplings with the q^2 dependence of the propagators accounted for, appear quite different at first sight from the latter (apart from the q^2 dependence which is assumed to be absent), especially since the anomalous couplings are written down in terms of the Levi-Civita symbols. At first instance the complete mapping has not been possible because in some cases, in the anomalous TGC sector, the basis chosen has been one that involves the Levi-Civita symbol (CP conserving case). The conventional treatment of contact interactions does not involve this symbol. However, it is possible through the use of Dirac matrix identities to choose an equivalent basis for the contact interactions as well, which could lead to a direct identification. We have studied the structures in detail and uncovered these relations so as to establish the correspondence. We have found that apart from the contact interactions studied earlier [28–30], a coupling containing three Dirac matrices is also required. The form factor containing the three Dirac matrices was introduced in [31] and the authors have also pointed out that this form factor receives a contribution from a dimension-8 operator of the form $\bar{l}\gamma^\mu l \epsilon_{\mu\nu\sigma\tau} D^\nu B^{\sigma\lambda} B_\lambda^\tau$, which is CP even and was considered earlier in Ref. [32].

In order to make contact with experiment, it is important to ask what the contributions of the TGCs at leading order would be to the diboson distribution, in the

²We have scaled the couplings by the factor 1/2 in case of h_i^Z and $1/(4s_W c_W)$ in case of h_i^γ of Ref. [5] for reasons to be explained in the next section.

presence of the two kinds of polarization. We study this using realistic degrees of polarization, and with the design luminosity at the various proposed ILC energies. We limit ourselves to centre-of-mass energy of 800 GeV along with an integrated luminosity of 500 fb^{-1} . Since the BSM contribution from the contact interactions or the effective couplings can be measured as deviations from the SM predictions in various kinematic distributions, we have carried out a thorough numerical analyses by the construction of various asymmetries. In particular the effect of beam polarization has been concentrated upon. In our previous work [17, 33], we were concerned only with the dimension-six CP-violating operators. Explicit distributions in the presence of longitudinal polarization (LP) and transverse polarization (TP) were obtained for this case. However, such an analysis has not been performed for the dimension-eight CP-violating operator, nor for any of the CP-conserving cases, at least not in the forms discussed in these references. One of the aims of this work is to obtain such distributions so as to set the stage for a thorough comparison with the types of distributions obtained with the contact interactions.

The layout of the paper is as follows. The process $e^+e^- \rightarrow Z\gamma$ is discussed in Sec. 2, which is divided into three subsections. We list in Sec. 2.1 the most general $Z\gamma V^*$ coupling, where $V = Z, \gamma$ and present the distributions in the presence of the anomalous couplings with polarized beams, both TP and LP. The new physics effect in the form of the contact interactions will be discussed in Sec. 2.2 and the mapping of contact interactions with triple gauge boson couplings is addressed in Sec. 2.3. The CPT properties of the different anomalous couplings are discussed in Sec. 3. In Sec. 4 we discuss how angular asymmetries may be constructed which could be used to get information on the couplings. We do a full numerical analysis on the anomalous couplings and give limits on those in Sec. 5. Finally we conclude in Sec. 6. The Appendix A discusses the reduction of the anomalous TGCs with the Levi-Civita symbol to an equivalent basis of the contact interactions.

2 Formalism for the process $e^+e^- \rightarrow Z\gamma$

In this section we discuss the properties of the process

$$e^+(p_+, s_+) + e^-(p_-, s_-) \rightarrow Z(k_Z, h_Z) + \gamma(k_1, h_\gamma), \quad (2.1)$$

where h_γ can take values ± 1 and the value for h_Z can be ± 1 and 0. In Fig. 1, we show the different diagrams which contribute to neutral gauge boson pair production. The first two diagrams (*a* and *b*) show the leading contribution coming from the standard model *t*- and *u*-channel electron exchanges. The new-physics effect in the form of anomalous TGCs due to the *s*-channel *Z* and γ exchanges is shown in the third diagram (*c*), which will be discussed in detail in Sec. 2.1. The effect due to contact interactions is shown in the final diagram (*d*), and will be the matter of discussion

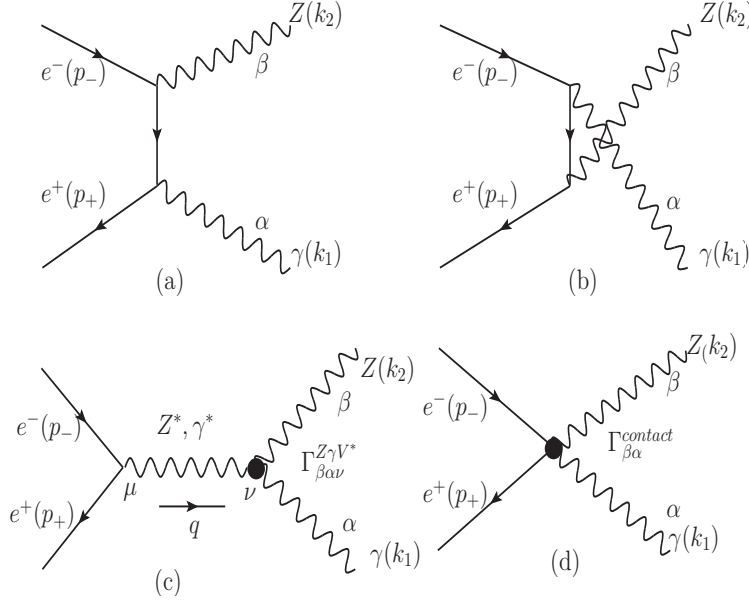


Figure 1. Feynman diagrams contributing to the neutral gauge boson production. Diagrams (a) and (b) are the SM contributions. Diagram (c) corresponds to contributions from the anomalous $Z\gamma Z$ and $Z\gamma\gamma$ couplings and diagram (d) corresponds to contribution from the contact interactions.

in the upcoming Sec. 2.2. In the final subsection 2.3 we present a detailed discussion of the TGCs in terms of the framework of contact interactions.

2.1 BSM physics with anomalous triple gauge boson couplings

The $Z\gamma$ production may have a contribution from the anomalous $Z\gamma Z^*$ or $Z\gamma\gamma^*$ couplings through the s channel, where Z, γ are on shell, while Z^*/γ^* is off shell. Since we neglect the electron mass, when the off-shell photon or Z couples to fermions, the corresponding current is conserved. Assuming $U(1)_{em}$ gauge invariance and Lorentz invariance, the most general anomalous $Z\gamma V$ coupling, where $V = Z^*, \gamma^*$ is given by

$$\begin{aligned} \Gamma_{\beta\alpha\nu}^{Z\gamma Z^*}(k_2, k_1, q) = & \frac{e(s - m_Z^2)}{2m_Z^2} \left\{ h_1^Z (k_{1\nu}g_{\beta\alpha} - k_{1\beta}g_{\nu\alpha}) + \frac{h_2^Z}{m_Z^2} q_\beta (q \cdot k_1 g_{\nu\alpha} - k_{1\nu}q_\alpha) \right. \\ & + h_3^Z \epsilon_{\nu\beta\alpha\rho} k_1^\rho + \frac{h_4^Z}{m_Z^2} q_\beta \epsilon_{\nu\alpha\rho\sigma} q^\rho k_1^\sigma \\ & \left. + \frac{h_5^Z}{2m_Z^2} [(s - m_Z^2)\epsilon_{\alpha\nu\beta\sigma}(k_2 + q)^\sigma - 4k_{2\alpha}\epsilon_{\nu\beta\tau\sigma}k_1^\tau q^\sigma] \right\}. \end{aligned} \quad (2.2)$$

$$\begin{aligned} \Gamma_{\beta\alpha\nu}^{Z\gamma\gamma^*}(k_2, k_1, q) = & \frac{es}{4s_W c_W m_Z^2} \left\{ h_1^\gamma (k_{1\nu}g_{\beta\alpha} - k_{1\beta}g_{\nu\alpha}) + \frac{h_2^\gamma}{m_Z^2} q_\beta (q \cdot k_1 g_{\nu\alpha} - k_{1\nu}q_\alpha) \right. \\ & \left. + h_3^\gamma \epsilon_{\nu\beta\alpha\rho} k_1^\rho + \frac{h_4^\gamma}{m_Z^2} q_\beta \epsilon_{\nu\alpha\rho\sigma} q^\rho k_1^\sigma \right\}. \end{aligned} \quad (2.3)$$

We note that the coupling $\Gamma_{\beta\alpha\nu}^{Z\gamma V^*}$ was first written down in [5]. However, [5] did not have the h_5^Z term. The unusual anomalous $Z\gamma Z^*$ vertex in the h_5^Z term, to our knowledge, has not been noted in the literature. Surprisingly, it does not have a $Z\gamma\gamma^*$ counterpart.

We have scaled the coupling constants by a factor of $1/2$ in case of $\Gamma^{Z\gamma Z^*}$ and $1/(4s_W c_W)$ in case of $\Gamma^{Z\gamma\gamma^*}$, in relation to those in [5]. This has been done to effect a simple comparison with the contact interactions case, where such factors are already absorbed into the definition of the relevant couplings. The choice is to either rescale the h_i^V terms of [5] or to rescale the contact terms of [28, 29], and we choose the former.

The effective Lagrangian generating the vertices of Eq. (2.2) is given by

$$\begin{aligned}\mathcal{L}^{Z\gamma Z^*} = & \frac{e}{2} \left\{ \frac{h_1^Z}{m_Z^2} (\partial^\sigma Z_{\sigma\nu}) Z_\alpha F^{\nu\alpha} + \frac{h_2^Z}{m_Z^4} [\partial_\beta \partial_\alpha (\Box + m_Z^2) Z_\nu] Z^\beta F^{\nu\alpha} \right. \\ & + \frac{h_3^Z}{m_Z^2} (\partial_\sigma Z^{\sigma\rho}) Z^\beta \tilde{F}_{\rho\beta} + \frac{h_4^Z}{2m_Z^4} [(\Box + m_Z^2) \partial^\sigma Z^{\rho\beta}] Z_\sigma \tilde{F}_{\rho\beta} \\ & \left. + \frac{h_5^Z}{m_Z^4} (\partial^\tau F^{\alpha\lambda}) \tilde{Z}_{\alpha\beta} \partial_\tau \partial_\lambda Z^\beta \right\},\end{aligned}\quad (2.4)$$

whereas the Lagrangian generating the vertices of Eq. (2.3) is given by

$$\begin{aligned}\mathcal{L}^{Z\gamma\gamma^*} = & \frac{e}{4s_W c_W} \left\{ \frac{h_1^\gamma}{m_Z^2} (\partial^\sigma F_{\sigma\nu}) Z_\alpha F^{\nu\alpha} + \frac{h_2^\gamma}{m_Z^4} [\partial_\beta \partial_\alpha \partial^\rho F_{\rho\nu}] Z^\beta F^{\nu\alpha} \right. \\ & \left. + \frac{h_3^\gamma}{m_Z^2} (\partial_\sigma F^{\sigma\rho}) Z^\beta \tilde{F}_{\rho\beta} + \frac{h_4^\gamma}{2m_Z^4} [\Box \partial^\sigma F^{\rho\beta}] Z_\sigma \tilde{F}_{\rho\beta} \right\}.\end{aligned}\quad (2.5)$$

Here,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu; \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \quad (2.6)$$

and

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}; \quad \tilde{Z}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} Z^{\alpha\beta}. \quad (2.7)$$

The matrix element from the SM t - and u -channel electron exchanges, and the anomalous coupling introduced by the vertices of Eqs. (2.2) and (2.3), which introduce respectively diagrams with s -channel Z and γ exchanges, is given by

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4, \quad (2.8)$$

h_i^V	$\Gamma_{\alpha\beta}^{Z,\gamma}$
$h_1^{\gamma,Z}$	$\gamma^\alpha k_1^\beta - g^{\alpha\beta} \not{k}_1$
$h_2^{\gamma,Z}$	$k_1^\beta (\not{k}_1 k_2^\alpha - \gamma^\alpha \frac{s-m_Z^2}{2})$
$h_3^{\gamma,Z}$	$\gamma_\nu \epsilon^{\alpha\beta\nu k_1}$
$h_4^{\gamma,Z}$	$-\gamma_\nu k_1^\beta \epsilon^{\alpha\nu k_1 q}$
h_5^Z	$\gamma_\nu \left(2k_2^\alpha \epsilon^{\beta k_2 \nu q} + \frac{m_Z^2 - s}{2} (\epsilon^{\alpha\beta k_2 \nu} + \epsilon^{\alpha\beta q \nu}) \right)$

Table 1. The two-index object $\Gamma_{\alpha\beta}^{Z,\gamma}$ obtained by contracting the three-index object $\Gamma_{\beta\alpha\nu}^{Z\gamma V^*}(k_2, k_1, q)$ with $\frac{(-\gamma_\nu + \not{q} q^\nu / m_Z^2)}{q^2 - m_Z^2}$ in case of the Z and $\frac{-\gamma_\nu}{q^2}$ in case of γ . A factor $(g_V - g_A \gamma_5)$ has to be multiplied on the right for all the $\Gamma_{\alpha\beta}^Z$ terms. An overall factor of m_Z^{-2} has to be included for the $h_1^{Z,\gamma}$ and $h_3^{Z,\gamma}$ terms, and a factor m_Z^{-4} for the rest.

where

$$\begin{aligned}
\mathcal{M}_1 &= \frac{e^2}{4c_W s_W} \bar{v}(p_+) \not{\epsilon}(k_2) (g_V - g_A \gamma_5) \frac{1}{\not{p}_- - \not{k}_1} \not{\epsilon}(k_1) u(p_-), \\
\mathcal{M}_2 &= \frac{e^2}{4c_W s_W} \bar{v}(p_+) \not{\epsilon}(k_1) \frac{1}{\not{p}_- - \not{k}_2} \not{\epsilon}(k_1) (g_V - g_A \gamma_5) u(p_-), \\
\mathcal{M}_3 &= \frac{ie}{2c_W s_W} \bar{v}(p_+) \gamma_\mu (g_V - g_A \gamma_5) u(p_-) \frac{(-g^{\mu\nu} + q^\mu q^\nu / m_Z^2)}{q^2 - m_Z^2} \Gamma_{\beta\alpha\nu}^{Z\gamma Z^*}(k_2, k_1, q) \epsilon^\alpha(k_1) \epsilon^\beta(k_2), \\
\mathcal{M}_4 &= ie \bar{v}(p_+) \gamma_\mu u(p_-) \frac{(-g^{\mu\nu})}{q^2} \Gamma_{\beta\alpha\nu}^{Z\gamma\gamma^*}(k_2, k_1, q) \epsilon^\alpha(k_1) \epsilon^\beta(k_2).
\end{aligned} \tag{2.9}$$

Here, the vector and axial-vector couplings of the Z to the electron are given by

$$g_V = -1 + 4s_W^2, \quad g_A = -1, \tag{2.10}$$

where $c_W = \cos \theta_W$, $s_W = \sin \theta_W$, θ_W being the weak mixing angle.

The three-index object $\Gamma_{\beta\alpha\nu}^{Z\gamma V^*}(k_2, k_1, q)$ is effectively contracted with $-\gamma_\nu/q^2$ in case of γ and $(-\gamma_\nu + \not{q} q^\nu / m_Z^2)/(q^2 - m_Z^2)$ in case of Z boson, which yields a convenient two-index object which we denote as $\Gamma_{\alpha\beta}^{Z,\gamma}$. We now list in Table 1 the various terms in $\Gamma_{\alpha\beta}^{Z,\gamma}$ in a much simplified form after dealing with the redundancies, and after dropping \not{q} terms which vanish (in the limit of vanishing electron mass) on using the Dirac equation.

When the e^- and e^+ beams have longitudinal polarizations P_L and \bar{P}_L , we obtain the differential cross section for the process (1) to be

$$\frac{d\sigma}{d\Omega_L} = \mathcal{B}_L (1 - P_L \bar{P}_L) \left[\mathcal{A}_L \frac{1}{\sin^2 \theta} \left(1 + \cos^2 \theta + \frac{4\bar{s}}{(\bar{s} - 1)^2} \right) + C_L \right], \tag{2.11}$$

where

$$\bar{s} \equiv \frac{s}{m_Z^2}, \quad \mathcal{B}_L = \frac{\alpha^2}{16 \sin^2 \theta_W m_W^2 \bar{s}} \left(1 - \frac{1}{\bar{s}} \right), \tag{2.12}$$

Coupling	Coefficient
$\text{Im } h_1^Z$	$-\frac{1}{2}\mathcal{A}_L(\bar{s}-1)\cos\theta$
$\text{Im } h_1^\gamma$	$-\frac{1}{2}(\bar{s}-1)(g_V - Pg_A)\cos\theta$
$\text{Im } h_2^Z$	$\frac{1}{4}\mathcal{A}_L\bar{s}(\bar{s}-1)\cos\theta$
$\text{Im } h_2^\gamma$	$\frac{1}{4}\bar{s}(\bar{s}-1)(g_V - Pg_A)\cos\theta$
$\text{Re } h_3^Z$	$-\frac{1}{2}(\bar{s}+1)(2g_Vg_A - P(g_V^2 + g_A^2))$
$\text{Re } h_3^\gamma$	$-\frac{1}{2}(\bar{s}+1)(g_A - Pg_V)$
$\text{Re } h_4^Z$	$\frac{1}{4}\bar{s}(\bar{s}-1)(2g_Vg_A - P(g_V^2 + g_A^2))$
$\text{Re } h_4^\gamma$	$\frac{1}{4}\bar{s}(\bar{s}-1)(g_A - Pg_V)$
$\text{Re } h_5^Z$	$-\frac{1}{4}(1+6\bar{s}+\bar{s}^2)(2g_Vg_A - P(g_V^2 + g_A^2))$

Table 2. The coefficients L_i^V of individual new couplings in the expression for the longitudinal polarization dependent part C_L , Eq. (2.15), of the cross section

with

$$P = \frac{P_L - \bar{P}_L}{1 - P_L \bar{P}_L}, \quad (2.13)$$

$$\mathcal{A}_L = (g_V^2 + g_A^2 - 2Pg_Vg_A), \quad (2.14)$$

$$C_L = \sum_{V=Z,\gamma} \left[\sum_{i=1}^2 (\text{Im } h_i^V) L_i^V + \sum_{i=3}^4 (\text{Re } h_i^V) L_i^V \right] + (\text{Re } h_5^Z) L_5^Z. \quad (2.15)$$

We choose the convention that P_L, \bar{P}_L are negative (positive) for left-handed (right-handed) polarization. C_L is a linear combination of the couplings h_i^V , ($i = 1, \dots, 5$), where $V = Z, \gamma$ for $i = 1 - 4$. We list in Table 2 the coefficient of each coupling L_i^V in the expression for C_L , Eq. (2.15) against the coupling.

The differential cross section for transverse polarizations P_T and \bar{P}_T of e^- and e^+ is given by

$$\frac{d\sigma}{d\Omega_T} = \mathcal{B}_T \left[\mathcal{A}_T \frac{1}{\sin^2\theta} \left(1 + \cos^2\theta + \frac{4\bar{s}}{(\bar{s}-1)^2} + P_T \bar{P}_T \frac{g_V^2 - g_A^2}{g_V^2 + g_A^2} \sin^2\theta \cos 2\phi \right) + C_T \right], \quad (2.16)$$

where \bar{s} is as defined before,

$$\mathcal{B}_T = \frac{\alpha^2}{16 \sin^2\theta_W m_W^2 \bar{s}} \left(1 - \frac{1}{\bar{s}} \right), \quad (2.17)$$

$$C_T = \sum_{V=Z,\gamma} \left\{ \sum_{i=1}^4 \left[(\text{Im } h_i^V) T_i^{V,I} + (\text{Re } h_i^V) T_i^{V,R} \right] \right\} + (\text{Im } h_5^Z) T_5^{Z,I} + (\text{Re } h_5^Z) T_5^{Z,R} \quad (2.18)$$

with $\mathcal{A}_T = (g_V^2 + g_A^2)$. C_T in Eq. (2.18) is a linear combination of the couplings and $V = Z, \gamma$, and the non-vanishing coefficients $T_i^{V,I}$ and $T_i^{V,R}$ of the various couplings in C_T are presented in Tables 3 and 4.

Coupling	Coefficient
Im h_1^Z	$-\frac{1}{2}(\bar{s} - 1) \cos \theta (g_V^2 + g_A^2 - (g_V^2 - g_A^2) P_T \bar{P}_T \cos 2\phi)$
Im h_1^γ	$-\frac{1}{2}(\bar{s} - 1) g_V \cos \theta (1 - P_T \bar{P}_T \cos 2\phi)$
Im h_2^Z	$\frac{1}{4}\bar{s}(\bar{s} - 1) \cos \theta (g_V^2 + g_A^2 - (g_V^2 - g_A^2) P_T \bar{P}_T \cos 2\phi)$
Im h_2^γ	$\frac{1}{4}\bar{s}(\bar{s} - 1) g_V \cos \theta (1 - P_T \bar{P}_T \cos 2\phi)$
Im h_3^Z	$\frac{1}{2}(\bar{s} - 1)(g_V^2 - g_A^2) P_T \bar{P}_T \sin 2\phi$
Im h_3^γ	$\frac{1}{2}(\bar{s} - 1) g_V P_T \bar{P}_T \sin 2\phi$
Im h_4^Z	$-\frac{1}{4}\bar{s}(\bar{s} - 1)(g_V^2 - g_A^2) P_T \bar{P}_T \sin 2\phi$
Im h_4^γ	$-\frac{1}{4}\bar{s}(\bar{s} - 1) g_V P_T \bar{P}_T \sin 2\phi$
Im h_5^Z	$\frac{1}{4}(\bar{s}^2 - 1)(g_V^2 - g_A^2) P_T \bar{P}_T \sin 2\phi$

Table 3. The coefficients $T_i^{V,I}$ of the imaginary part of the individual new couplings in the expression for the transverse polarization dependent part C_T , Eq. (2.18), of the cross section. Only the non-zero entries are listed here.

Coupling	Coefficient
Re h_1^γ	$\frac{1}{2}(\bar{s} - 1) g_A \cos \theta P_T \bar{P}_T \sin 2\phi$
Re h_2^γ	$-\frac{1}{4}\bar{s}(\bar{s} - 1) g_A \cos \theta P_T \bar{P}_T \sin 2\phi$
Re h_3^Z	$-(\bar{s} + 1) g_A g_V$
Re h_3^γ	$-\frac{1}{2} g_A ((\bar{s} + 1) + (\bar{s} - 1) P_T \bar{P}_T \cos 2\phi)$
Re h_4^Z	$\frac{1}{2}\bar{s}(\bar{s} - 1) g_V g_A$
Re h_4^γ	$\frac{1}{4}\bar{s}(\bar{s} - 1) g_A (1 + P_T \bar{P}_T \cos 2\phi)$
Re h_5^Z	$-\frac{1}{2} g_A g_V (1 + 6\bar{s} + \bar{s}^2)$

Table 4. The coefficients $T_i^{V,R}$ of the real part of the individual new couplings in the expression for the transverse polarization dependent part C_T , Eq. (2.18), of the cross section. Only the non-zero entries are listed here.

We have kept the anomalous terms up to leading order since they are expected to be small. In the above expressions, θ is the angle between the photon and e^- direction, with the e^- direction chosen as the z axis. The azimuthal angle between the photon and the electron momentum direction is chosen to be ϕ . The transverse polarization of the electron is chosen along the x axis, whereas the positron polarization direction is chosen parallel to the electron polarization direction.

It can be seen from Tables 2, 4 that $\text{Re } h_{1,2}^Z$ does not contribute to the distribution, with or without beam polarization. The question of isolating $\text{Re } h_1^Z$ to leading order was recently addressed by us [17], where we pointed out that it would be possible to fingerprint this anomalous coupling if the final-state spins are resolved. Analogously the contribution of $\text{Re } h_2^Z$ can be studied by analyzing the spin of the final-state particles. Tables 2, 3 and 4 also show that some of the anomalous cou-

plings either depend on LP or TP or both, like the anomalous couplings $\text{Re } h_{1,2}^\gamma$, $\text{Im } h_{3,4}^{Z,\gamma}$ only give contributions in the presence of TP. It will therefore be possible to map the correspondence between these anomalous form factors and the contact interactions by studying the behaviour of the distributions in the presence of different beam polarizations.

In the next subsection, we turn to the issue of parametrizing the BSM physics in terms of contact interactions, viz., ones where all the new physics is integrated out, and only kinematic information is encoded in the vectors on hand. The case of anomalous TGC can be mapped to this, after accounting for the (trivial) momentum dependence coming from the propagators. The non-trivial kinematic structure due to anomalous TGC would form a proper subset of the general considerations, which we seek to establish. The two-index object introduced earlier, provides the required bridge to do this.

2.2 BSM physics in the form of contact interactions

We now introduce BSM physics arising from contact $e^+e^- \rightarrow Z\gamma$ interactions as shown in the Feynman diagram (d) of Fig. 1. The corresponding matrix element for the process of Eq. (2.1) in the presence of contact interactions will be of the form :

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_{\text{contact}}, \quad (2.19)$$

where $\mathcal{M}_{1,2}$ are defined before in Eq. 2.9, and

$$\mathcal{M}_{\text{contact}} = \frac{ie^2}{4c_W s_W} \bar{v}(p_+) \Gamma'_{\alpha\beta} u(p_-) \epsilon^\alpha(k_1) \epsilon^\beta(k_2) \quad (2.20)$$

The vertex factor $\Gamma_{\alpha\beta}$ of contact interactions was studied earlier in [28, 29, 31], where it was parametrized in the form :

$$\Gamma_{\alpha\beta}^{\text{contact}} = \frac{ie^2}{4c_W s_W} \Gamma'_{\alpha\beta}, \quad (2.21)$$

where

$$\begin{aligned} \Gamma'_{\alpha\beta} = & \left\{ \frac{1}{m_Z^4} ((v_1 + a_1 \gamma_5) \gamma_\beta (2p_{-\alpha}(p_+ \cdot k_1) - 2p_{+\alpha}(p_- \cdot k_1)) + \right. \\ & ((v_2 + a_2 \gamma_5) p_{-\beta} + (v_3 + a_3 \gamma_5) p_{+\beta}) (\gamma_\alpha 2p_- \cdot k_1 - 2p_{-\alpha} \not{k}_1) + \\ & ((v_4 + a_4 \gamma_5) p_{-\beta} + (v_5 + a_5 \gamma_5) p_{+\beta}) (\gamma_\alpha 2p_+ \cdot k_1 - 2p_{+\alpha} \not{k}_1) + \\ & \left. \frac{1}{m_Z^2} ((v_6 + a_6 \gamma_5) (\gamma_\alpha k_{1\beta} - \not{k}_1 g_{\alpha\beta}) + (v_7 + a_7 \gamma_5) \not{k}_1 \gamma_\alpha \gamma_\beta) \right\}. \end{aligned} \quad (2.22)$$

The above is the most general form consistent with Lorentz and gauge invariance, and written in terms of an odd number of γ matrices, so that chirality is conserved by the vertex.

When the only BSM interactions present are the triple-gauge boson couplings shown in Eqs. (2.2) and (2.3), the vertex factor $\Gamma'_{\alpha\beta}$ is effectively the sum of the $\Gamma_{\alpha\beta}^{Z,\gamma}$ terms of Table 1 appropriately weighted:

$$\Gamma'_{\alpha\beta} = \sum_{i=1,3} m_Z^{-2} [h_i^\gamma \Gamma_{\alpha\beta}^{\gamma,i} + h_i^Z \Gamma_{\alpha\beta}^{Z,i} (g_V - \gamma_5 g_A)] + \sum_{i=2,4,5} m_Z^{-4} [h_i^\gamma \Gamma_{\alpha\beta}^{\gamma,i} + h_i^Z \Gamma_{\alpha\beta}^{Z,i} (g_V - \gamma_5 g_A)] \quad (2.23)$$

Of course, it is always possible that there are further interactions present which do not contribute to the triple-gauge couplings, but contribute in the form of contact interactions. One of our aims here is to make a correspondence between the form factors v_i , a_i written in the contact interactions and those in the triple-gauge boson vertices. The distributions arising from the new couplings (with the exception of v_7 and a_7) in the presence of both longitudinal and transverse polarization were given in [28, 29]. We would also like to compare these distributions with those obtained in the previous section.

The contributions of the new contact interactions to the cross section with longitudinal and transverse polarizations of the beams, as defined respectively by C_L and C_T of Eqs. (2.11) and (2.16), are given by

$$C_L = \frac{1}{4} \left\{ \sum_{i=1}^7 ((g_V - P g_A) \text{Im} v_i + (g_A - P g_V) \text{Im} a_i) X_i \right\}, \quad (2.24)$$

and

$$C_T = \frac{1}{4} \left\{ \sum_{i=1}^7 (g_V \text{Im} v_i + g_A \text{Im} a_i) X_i + P_T \overline{P}_T \times \sum_{i=1}^7 ((g_V \text{Im} v_i - g_A \text{Im} a_i) \cos 2\phi + (g_A \text{Re} v_i - g_V \text{Re} a_i) \sin 2\phi) Y_i \right\}. \quad (2.25)$$

X_i and Y_i ($i = 1, \dots, 7$) are listed in Table 2.2.

In case of the contact interactions it is seen that, with the exception of $v_{6,7}$ and $a_{6,7}$, the anomalous form factors either contribute to the transverse polarization dependent part, or to the longitudinal polarization dependent and polarization independent parts of the differential cross section, but not both. The anomalous form factors $v_{6,7}$ and $a_{6,7}$, on the other hand, contribute to both.

2.3 Reduction of anomalous TGC interactions to contact type interactions

In order to make a correspondence between the two approaches, we compare the matrix elements of Eq. (2.9) and Eq. (2.19), using Eq. (2.23) and using the forms of $\Gamma_{\alpha\beta}^{Z,\gamma}$ with the Levi-Civita tensor, if any, rewritten using the results of the Appendix A.

i	X_i	Y_i
1	$2\bar{s}(\bar{s} + 1)$	0
2	$-\bar{s}(\bar{s} - 1)(\cos \theta - 1)$	0
3	0	$\bar{s}(\bar{s} - 1)(\cos \theta - 1)$
4	0	$\bar{s}(\bar{s} - 1)(\cos \theta + 1)$
5	$-\bar{s}(\bar{s} - 1)(\cos \theta + 1)$	0
6	$-2(\bar{s} - 1) \cos \theta$	$2(\bar{s} - 1) \cos \theta$
7	$2(\bar{s} - 1)(1 + \cos \theta) + 4$	$-2(\bar{s} - 1)(1 + \cos \theta)$

Table 5. The contribution of the new couplings to the polarization independent and dependent parts of the cross section.

On equating coefficients of the independent γ -matrix and tensor combinations, we get the relations

$$v_1 = -2ih_Z^5 g_A, \quad (2.26)$$

$$v_2 + v_3 + v_4 + v_5 = -2(h_2^\gamma + h_2^Z g_V), \quad (2.27)$$

$$v_2 + v_3 - v_4 - v_5 = 2ih_4^Z g_A, \quad (2.28)$$

$$v_2 - v_3 - v_4 + v_5 = 0, \quad (2.29)$$

$$v_2 - v_3 + v_4 - v_5 = 4ih_5^Z g_A, \quad (2.30)$$

$$v_6 = h_1^\gamma + h_1^Z g_V - ih_3^Z g_A + ih_5^Z g_A(s - m_Z^2)/(2m_Z^2), \quad (2.31)$$

$$v_7 = i(-h_3^Z g_A + h_5^Z g_A(s - m_Z^2)/(2m_Z^2)), \quad (2.32)$$

and

$$a_1 = -2ih_5^Z g_V, \quad (2.33)$$

$$a_2 + a_3 + a_4 + a_5 = -2h_2^Z g_A, \quad (2.34)$$

$$a_2 + a_3 - a_4 - a_5 = 2i(h_4^Z g_V + h_4^\gamma), \quad (2.35)$$

$$a_2 - a_3 - a_4 + a_5 = 0, \quad (2.36)$$

$$a_2 - a_3 + a_4 - a_5 = 4ih_5^Z g_V, \quad (2.37)$$

$$a_6 = (h_1^Z g_A - i(h_3^\gamma + h_3^Z g_V) + ih_5^Z g_V(s - m_Z^2)/(2m_Z^2)), \quad (2.38)$$

$$a_7 = i(-h_3^Z g_V - h_3^\gamma + h_5^Z g_V(s - m_Z^2)/(2m_Z^2)). \quad (2.39)$$

These may be solved for v_i , a_i in terms of the h_i^V . The above relations hold at the amplitude level. In turn, the distributions generated by the v_i , a_i of the contact interactions would be indistinguishable from the distribution generated by the TGCs with coefficients obeying these equations. The TGCs being less in number than the contact interactions, when the contact interactions come from TGCs, they obey constraints among themselves. These constraints can then be a test of whether the TGCs describe the full new physics or not.

3 Discrete symmetries of the BSM interactions

In order to study the properties of the different TGCs, by the construction of different asymmetries, we need to first understand the CP properties of various terms in the differential cross section. For completeness, we now provide a brief recapitulation of the discussion provided in the case of contact interactions [28, 29], which we now extend in the case of anomalous TGCs. Firstly, we consider the case of TP, for which we note the following relations:

$$\vec{P} \cdot \vec{k}_1 = \frac{\sqrt{s}}{2} |\vec{k}_1| \cos \theta, \quad (3.1)$$

$$(\vec{P} \times \vec{s}_- \cdot \vec{k}_1)(\vec{s}_+ \cdot \vec{k}_1) + (\vec{P} \times \vec{s}_+ \cdot \vec{k}_1)(\vec{s}_- \cdot \vec{k}_1) = \frac{\sqrt{s}}{2} |\vec{k}_1|^2 \sin^2 \theta \sin 2\phi, \quad (3.2)$$

$$(\vec{s}_- \cdot \vec{s}_+)(\vec{P} \cdot \vec{P} \vec{k}_1 \cdot \vec{k}_1 - \vec{P} \cdot \vec{k}_1 \vec{P} \cdot \vec{k}_1) - 2(\vec{P} \cdot \vec{P})(\vec{s}_- \cdot \vec{k}_1)(\vec{s}_+ \cdot \vec{k}_1) = -\frac{s}{4} |\vec{k}_1|^2 \sin^2 \theta \cos 2\phi. \quad (3.3)$$

In the above equations, $\vec{P} = \frac{1}{2}(\vec{p}_- - \vec{p}_+)$, where p_- is the momentum of the electron, and p_+ is the momentum of the positron. Moreover it is assumed that $\vec{s}_+ = \vec{s}_-$; taking $\vec{s}_+ = -\vec{s}_-$ would only give an overall negative sign for all the terms. Observing that the vector \vec{P} is C and P odd, that the photon momentum \vec{k}_1 is C even but P odd, and that the spin vectors \vec{s}_\pm are P even, and go into each other under C, we can immediately check that only the left-hand side (lhs) of Eq. (3.1) is CP odd, while the lhs of Eqs. (3.2) and (3.3) are CP even. Of all the above, only the lhs of (3.2) is odd under naive time reversal T.

Many of these features can be explicitly checked from Tables 3, 4: we see that the term $\cos \theta$ is accompanied by the CP violating couplings $h_1^Z, h_2^Z, h_1^\gamma, h_2^\gamma$, whereas the CP conserving couplings $h_3^Z, h_4^Z, h_5^Z, h_3^\gamma, h_4^\gamma, h_5^\gamma$ have no $\cos \theta$ dependence. It is known that invariance under CPT implies that terms with the right-hand side (rhs) of (3.1) by itself, or multiplying the rhs of Eq. (3.3) would occur with absorptive (imaginary) parts of the form factors, whereas the rhs of Eq. (3.1) multiplied by the rhs of Eq. (3.2) would appear with dispersive (real) parts of the form factors. Therefore the imaginary part of the CP-odd terms always come with a factor of $\cos \theta$ or $\cos \theta \cos 2\phi$ and the real parts are accompanied by the factor $\cos \theta \sin 2\phi$. Similarly the imaginary part of the CP-even terms, which has no $\cos \theta$ dependence always come with a factor of $\sin 2\phi$ and the real parts are either accompanied with the factor $\cos 2\phi$ or no θ, ϕ dependence. The CPT dependence of the different anomalous couplings are used to construct the different asymmetries to be proposed and discussed in the next section.

As discussed in the earlier work [28, 29], in case of the contact interactions (Sec. 2.2), the coefficients of the combinations of couplings $r_2 + r_5, r_3 + r_4$, and of the coupling $r_6, (r_i = v_i, a_i)$ have a pure $\cos \theta$ dependence and are CP odd. On the other hand, the coefficients of r_1 and of the remaining linearly independent combinations

$r_2 - r_5$, $r_3 - r_4$, ($r_i = v_i, a_i$) have no $\cos \theta$ dependence. These combinations have been isolated by considering the tensors accompanying the coefficients r_i . Keeping in mind the fact that under C $p_+ \leftrightarrow p_-$ and $k_{1,2} \leftrightarrow k_{1,2}$, these properties may be readily inferred from the form of the tensors for $i = 1, \dots, 6$. An analysis with the inclusion of r_7 is more complicated. By construction, the r_7 term has no straightforward transformation property under C. An analysis must include r_6 and r_7 jointly. Writing the r_6 and r_7 terms as $r_6 \mathcal{O}_6 + r_7 \mathcal{O}_7$, where \mathcal{O}_6 and \mathcal{O}_7 are Dirac operators sandwiched between spinors, we can rewrite these terms as

$$(r_6 - r_7) \mathcal{O}_6 + r_7 (\mathcal{O}_6 + \mathcal{O}_7)$$

It may be verified $(\mathcal{O}_6 + \mathcal{O}_7) \equiv (\gamma_\alpha k_{1\beta} - \not{k}_1 g_{\alpha\beta} + \not{k}_1 \gamma_\alpha \gamma_\beta)$ is CP even. We conclude that while r_6 accompanies a purely CP-odd operator \mathcal{O}_6 , the operator multiplying r_7 is in part CP odd (viz., $-\mathcal{O}_6$), and in part CP even ($\mathcal{O}_6 + \mathcal{O}_7$). Thus Eqs. (2.31) and (2.32) and the corresponding Eqs. (2.38) and (2.39) for the a 's are consistent with this, since $h_3^{Z,\gamma}$ and h_5^Z which are CP even contribute equally to r_6 and r_7 . This completes our discussion of the discrete symmetry properties of the BSM physics in the process.

In case of longitudinal polarization, apart from Eq. (3.1), there is another CP-odd quantity, viz.,

$$\frac{1}{2} (\vec{s}_- + \vec{s}_+) \cdot \vec{k}_1 = |\vec{k}_1| \cos \theta. \quad (3.4)$$

While this is also proportional to $\cos \theta$ like (3.1), it is expected to appear with a factor $(P_L - \overline{P}_L)$ multiplying it. It is also CPT odd, and would therefore occur with the absorptive parts of form factors. With all these considerations in view, we now embark on the task of constructing suitable asymmetries to isolate the anomalous TGCs which is the aim of the next section.

4 Angular asymmetries for anomalous TGCs

In earlier studies, several asymmetries were considered to isolate the effects of contact interactions. Since, in this work we do not extend that sector, except for the couplings v_7 and a_7 , we will be primarily concerned with the task of isolating the anomalous TGCs, which form the main focus of our study. Contact interactions have been brought in for making a correspondence and showing that TGCs do not exhaust all possibilities. The angular distributions defined in Tables 2, 3, 4, involve several different functions of θ and ϕ , such as $\sin 2\phi$, $\sin 2\phi \cos \theta$, $\sin 2\phi \sin \theta$, $\cos 2\phi$, $\cos 2\phi \cos \theta$ etc. We next formulate different angular asymmetries which can be used to determine or disentangle the different linear combinations of the anomalous couplings. For all our calculations we have assumed a cut-off θ_0 on the polar angle θ of the photon in the forward and backward directions in order to stay away from the

beam pipe. This cut-off may be chosen to optimize the sensitivity of the measurement.

We first present the case of transverse polarization where we have considered both CP-odd and CP-even asymmetries so as to determine the anomalous couplings. The asymmetries defined in general are an appropriate asymmetry in ϕ , $A_{i2}, i = 1, 2, 3, 4$, and the same ϕ asymmetry combined with a forward-backward asymmetry in $A_{i1}, i = 1, 2, 3$. The forward-backward asymmetry in A_{i1} isolates terms with a θ dependence of $\cos \theta$, i.e., it is a CP-odd asymmetry, whereas A_{i2} isolates θ dependence which is either trivial, or proportional to $\sin \theta$. The asymmetry A_{i2} is sensitive to the CP-even couplings. The CP-odd asymmetries are defined as follows³:

$$A_{11}(\theta_0) = \frac{1}{\sigma^{SM}} \sum_{n=0}^3 (-1)^n \left(\int_{-\cos \theta_0}^0 d \cos \theta - \int_0^{\cos \theta_0} d \cos \theta \right) \int_{\pi n/2}^{\pi(n+1)/2} d\phi \frac{d\sigma}{d\Omega}, \quad (4.1)$$

$$A_{21}(\theta_0) = \frac{1}{\sigma^{SM}} \sum_{n=0}^3 (-1)^n \left(\int_{-\cos \theta_0}^0 d \cos \theta - \int_0^{\cos \theta_0} d \cos \theta \right) \int_{\pi(2n-1)/4}^{\pi(2n+1)/4} d\phi \frac{d\sigma}{d\Omega}, \quad (4.2)$$

$$A_{31}(\theta_0) = \frac{1}{\sigma^{SM}} \left(\int_{-\cos \theta_0}^0 d \cos \theta - \int_0^{\cos \theta_0} d \cos \theta \right) \left(\int_{-\pi/4}^{\pi/4} d\phi \frac{d\sigma}{d\Omega} + \int_{3\pi/4}^{5\pi/4} d\phi \frac{d\sigma}{d\Omega} \right) \quad (4.3)$$

whereas the CP-even asymmetries are⁴

$$A_{12}(\theta_0) = \frac{1}{\sigma^{SM}} \sum_{n=0}^3 (-1)^n \int_{-\cos \theta_0}^{\cos \theta_0} d \cos \theta \int_{\pi n/2}^{\pi(n+1)/2} d\phi \frac{d\sigma}{d\Omega}, \quad (4.4)$$

$$A_{22}(\theta_0) = \frac{1}{\sigma^{SM}} \sum_{n=0}^3 (-1)^n \int_{-\cos \theta_0}^{\cos \theta_0} d \cos \theta \int_{\pi(2n-1)/4}^{\pi(2n+1)/4} d\phi \frac{d\sigma}{d\Omega}, \quad (4.5)$$

$$A_{32}(\theta_0) = \frac{1}{\sigma^{SM}} \int_{-\cos \theta_0}^{\cos \theta_0} d \cos \theta \left(\int_{-\pi/4}^{\pi/4} d\phi \frac{d\sigma}{d\Omega} + \int_{3\pi/4}^{5\pi/4} d\phi \frac{d\sigma}{d\Omega} \right), \quad (4.6)$$

with

$$\sigma^{SM} \equiv \sigma^{SM}(\theta_0) = \int_{-\cos \theta_0}^{\cos \theta_0} d \cos \theta \int_0^{2\pi} d\phi \frac{d\sigma_{SM}}{d\Omega}. \quad (4.7)$$

The choice of the asymmetries is such that each asymmetry is dependent on a particular form of angular dependence. For instance in the asymmetry A_{12} , only the terms proportional to $\sin 2\phi$ or $\sin 2\phi \sin \theta$ survive, whereas in case of A_{11} it is the $\sin 2\phi \cos \theta$ terms which survive. The terms proportional to $\sin 2\phi$ or $\sin 2\phi \sin \theta$ are CPT odd and appear with the imaginary part of the anomalous couplings whereas

³In case of contact interactions, A_{11} is proportional to $(\text{Re } r_6 - \text{Re } r_7)$, and $A_{21,31}$ are proportional to $(\text{Im } r_6 - \text{Im } r_7)$. We list them here as the coupling r_7 was not discussed in [28, 29]

⁴In case of contact interactions, A_{12} is proportional to $\text{Re } r_7$, and $A_{22,32}$ are proportional to $\text{Im } r_7$. We list them here as the coupling r_7 was not discussed in [28, 29]

the $\sin 2\phi \cos \theta$ terms are CPT even and appear with the real part of the anomalous couplings, as discussed in Sec. 3. The SM contribution to $A_{11,12}$ is zero, since, as can be seen from Eq. (2.16), it has no $\sin 2\phi$ terms. Therefore the observation of either of these asymmetries at the ILC will point towards contribution from anomalous couplings. Similarly A_{22} has terms proportional to $\cos 2\phi$ and $\cos 2\phi \sin \theta$ and A_{21} has $\cos 2\phi \cos \theta$ dependence. It can be argued like before that the SM contribution to A_{21} will be zero and A_{22} will occur with the real parts of the anomalous couplings whereas A_{21} will receive contribution from the imaginary parts. It can be checked that the other asymmetries $A_{31,32}$ contain terms which are not proportional to the transverse polarization.

We present below the dependence of the asymmetries on the various anomalous couplings. The CP-odd asymmetries are given by

$$A_{11}(\theta_0) = \mathcal{B}'_T g_A P_T \bar{P}_T (\bar{s} - 1) \cos^2 \theta_0 (\bar{s} \text{Re} h_2^\gamma - 2 \text{Re} h_1^\gamma), \quad (4.8)$$

$$A_{21}(\theta_0) = \mathcal{B}'_T P_T \bar{P}_T (-1 + \bar{s}) \cos^2 \theta_0 (2 \text{Im} h_1^Z (g_V^2 - g_A^2) + 2 \text{Im} h_1^\gamma g_V + (\text{Im} h_2^Z (g_A^2 - g_V^2) - g_V \text{Im} h_2^\gamma) \bar{s}), \quad (4.9)$$

$$A_{31}(\theta_0) = \frac{\mathcal{B}'_T}{4} (\bar{s} - 1) \cos^2 \theta_0 \{ -g_V (\pi - 2 P_T \bar{P}_T) (2 \text{Im} h_1^\gamma - \bar{s} \text{Im} h_2^\gamma) - [g_V^2 (\pi - 2 P_T \bar{P}_T) + g_A^2 (\pi + 2 P_T \bar{P}_T)] (2 \text{Im} h_1^Z - \bar{s} \text{Im} h_2^Z) \}, \quad (4.10)$$

and the CP-even asymmetries by

$$A_{12}(\theta_0) = 2 \mathcal{B}'_T P_T \bar{P}_T (\bar{s} - 1) (g_V (-2 \text{Im} h_3^\gamma + \bar{s} \text{Im} h_4^\gamma) + (\text{Im} h_5^Z (\bar{s} + 1) + 2 \text{Im} h_3^Z - \bar{s} \text{Im} h_4^Z) (g_A^2 - g_V^2)) \cos \theta_0, \quad (4.11)$$

$$A_{22}(\theta_0) = 2 \mathcal{B}'_T P_T \bar{P}_T \cos \theta_0 (4 (g_A^2 - g_V^2) + g_A (\bar{s} - 1) (\bar{s} \text{Re} h_4^\gamma - 2 \text{Re} h_3^\gamma)), \quad (4.12)$$

$$A_{32}(\theta_0) = -\frac{\mathcal{B}'_T}{2} \left[\{ 4 (g_V^2 (\pi + 2 P_T \bar{P}_T) + g_A^2 (\pi - 2 P_T \bar{P}_T)) + g_A \right. \\ \left. [-2 P_T \bar{P}_T (\bar{s} - 1) (\bar{s} \text{Re} h_4^\gamma - 2 \text{Re} h_3^\gamma) + \pi (-\bar{s} (\bar{s} - 1) \text{Re} h_4^\gamma + 2 (\bar{s} + 1) \text{Re} h_3^\gamma + 2 g_V [\text{Re} h_5^Z + 2 \text{Re} h_3^Z (\bar{s} + 1) + \bar{s} \{ \text{Re} h_4^Z (1 - \bar{s}) + \text{Re} h_5^Z (6 + \bar{s}) \}])] \} \right] \cos \theta_0 \\ + 4 \pi \mathcal{A}_T \frac{1 + \bar{s}^2}{(\bar{s} - 1)^2} \log \left(\frac{1 - \cos \theta_0}{1 + \cos \theta_0} \right) \Big], \quad (4.13)$$

where $\mathcal{B}'_T = \mathcal{B}_T / \sigma^{SM}(\theta_0)$, and

$$\sigma^{SM}(\theta_0) = 4 \pi \mathcal{A}_T \mathcal{B}_T \left[\frac{1 + \bar{s}^2}{(\bar{s} - 1)^2} \log \left(\frac{1 + \cos \theta_0}{1 - \cos \theta_0} \right) - \cos \theta_0 \right]. \quad (4.14)$$

We have also considered a CP-odd asymmetry in the presence of longitudinal polarization, which is proportional to $\cos \theta$. It is shown in Sec. 3, Eqs. (3.1), (3.4) that the term proportional to $\cos \theta$ is CPT odd and would therefore always occur with the imaginary part of the anomalous couplings. The asymmetry is a forward-backward asymmetry

$$A^{LP}(\theta_0) = \frac{1}{\sigma_{LP}^{SM}} \left(\int_{-\cos \theta_0}^0 d \cos \theta - \int_0^{\cos \theta_0} d \cos \theta \right) \int_0^{2\pi} d\phi \frac{d\sigma}{d\Omega}, \quad (4.15)$$

with the form

$$A^{LP}(\theta_0) = \frac{\mathcal{B}'_L \pi}{2} (\bar{s} - 1) \cos^2 \theta_0 \left(-2 \left[(g_V - P g_A) \text{Im} h_1^\gamma + \mathcal{A}_L \text{Im} h_1^Z \right] \right. \\ \left. + \left[(g_V - P g_A) \text{Im} h_2^\gamma + \mathcal{A}_L \text{Im} h_2^Z \right] \bar{s} \right), \quad (4.16)$$

where $\mathcal{B}'_L = \mathcal{B}_L / \sigma_{LP}^{SM}(\theta_0)$. In the presence of longitudinal polarization, \mathcal{B}_T is replaced by $\mathcal{B}_L(1 - P_L \bar{P}_L)$ and \mathcal{A}_T is replaced by \mathcal{A}_L in Eq. (4.14) for $\sigma_{LP}^{SM}(\theta_0)$.

In the next section we evaluate these asymmetries numerically and investigate what limits on couplings may be expected by an experimental study of the asymmetries.

5 Numerical analysis

The asymmetries listed above receive contributions from combinations of the couplings. Since the number of different types of terms in the angular distribution is not large, it will not be possible to disentangle the effects of all the anomalous couplings, without a full-fledged fit to the distributions. The presence of all of them at the same time will make the numerical analysis complicated. We have therefore estimated possible 90% CL limits on the couplings assuming only one coupling to be non-zero at a time. For our discussion we have assumed \sqrt{s} of 800 GeV, along with an integrated luminosity of 500 fb⁻¹. The magnitudes of electron and positron polarization are taken to be 0.8 and 0.6 respectively. When an asymmetry arises only in the presence of BSM the 90% CL limits on the coupling, denoted by \mathcal{C}_{lim} , is related to the value A of the generic asymmetry for unit value of the anomalous coupling by

$$\mathcal{C}_{lim} \equiv \frac{1.64}{|A| \sqrt{N_{SM}}}, \quad (5.1)$$

where N_{SM} is the number of SM events. The coefficient 1.64 may be obtained from statistical tables for hypothesis testing with one estimator; see, e.g., Table 36.1 of Ref. [34].

We present here our results for the best limits obtainable on the anomalous couplings from various asymmetries. Since the anomalous couplings with $\sin 2\phi$ dependence give non-zero contribution for the asymmetries $A_{11,12}$, we present our results for this case. Along with it we also consider the asymmetries A^{LP} , A_{31} , A_{32} . We show in Fig. 2 the SM cross section, with a cut-off θ_0 in the forward and backward directions, as a function of θ_0 .

In case of the longitudinal polarization, we have considered the forward-backward asymmetry. It can be seen from Eq. (4.16) that the SM contribution is equal to zero and the couplings which contribute are $\text{Im} h_{1,2}^\gamma$ and $\text{Im} h_{1,2}^Z$. The coefficient of $\text{Im} h_1^\gamma$ and $\text{Im} h_2^\gamma$ are dependent on the choice of beam polarization. For our choice of beam polarization, $P_L = -0.8$ and $\bar{P}_L = 0.6$, the coefficients $(g_V - P g_A)$ and \mathcal{A}_L

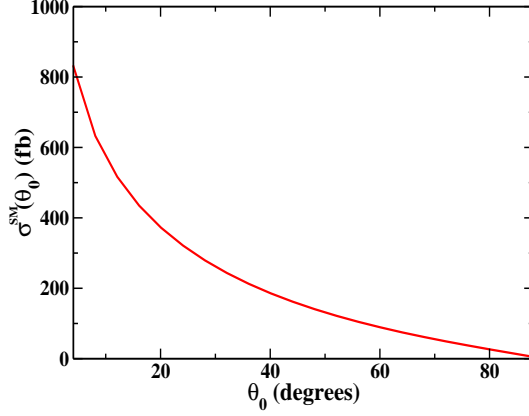


Figure 2. The SM cross section $\sigma^{SM}(\theta_0)$ defined in Eq. (4.14) as a function of the cut-off angle θ_0 at $\sqrt{s} = 800$ GeV.

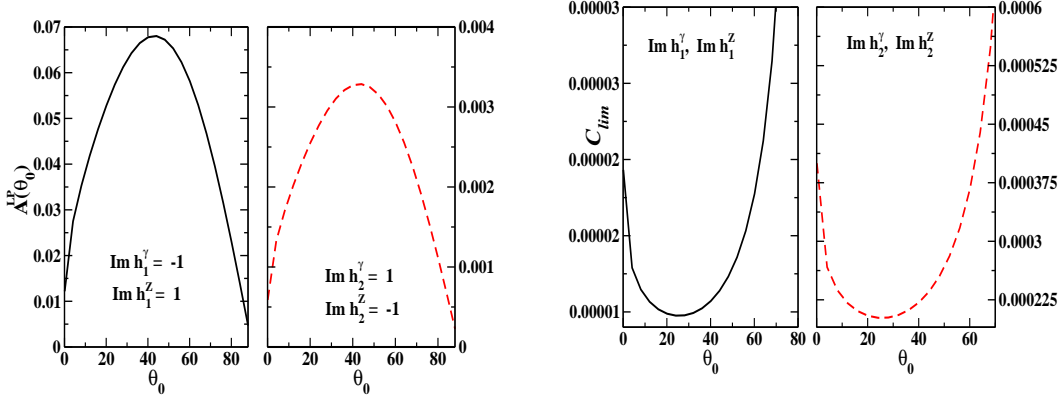


Figure 3. The asymmetry $A^{LP}(\theta_0)$ defined in Eq. (4.16) as a function of the cut-off angle θ_0 for the various couplings.

Figure 4. The 90% CL limits, Eq. (5.1) on the anomalous couplings from $A^{LP}(\theta_0)$, as a function of the cut-off angle θ_0 .

are almost the same apart from a minus sign. Therefore the behaviour of $|\text{Im } h_1^\gamma|$ and $|\text{Im } h_1^Z|$ will be the same. They will however behave differently with unpolarized beams but with less sensitivity. Fig. 3 shows the asymmetry $A^{LP}(\theta_0)$ as a function of the cut-off angle θ_0 , with the assumption of only one anomalous coupling being present at a time. We have next shown in Fig. 4 the 90% CL limits that can be obtained on these couplings from the asymmetry. It can be seen from Fig. 4 that the limit is almost independent of the cut-off angle θ_0 for the range $20^\circ < \theta_0 < 40^\circ$. We consider an optimal value of 30° , with the sensitivity on $\text{Im } h_1^{Z,\gamma}$ being 1×10^{-5} , and $\text{Im } h_2^{Z,\gamma}$ being 2.2×10^{-4} .

We next consider the asymmetries $A_{11,12}(\theta_0)$, which are dependent on a different set of couplings. We would like to repeat that the SM contribution to these asymmetries is zero. We plot in Figs. 5 and 6 the various asymmetries as a function of the

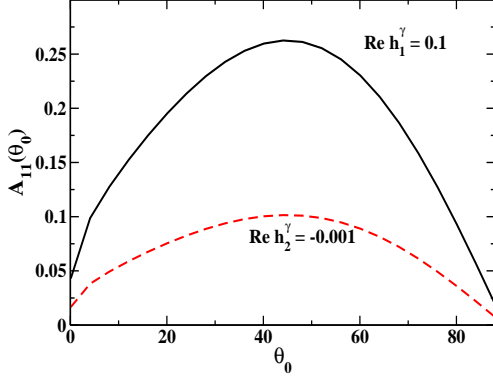


Figure 5. The asymmetry $A_{11}(\theta_0)$ defined in Eq. (4.8) as a function of the cut-off angle θ_0 for the various couplings.

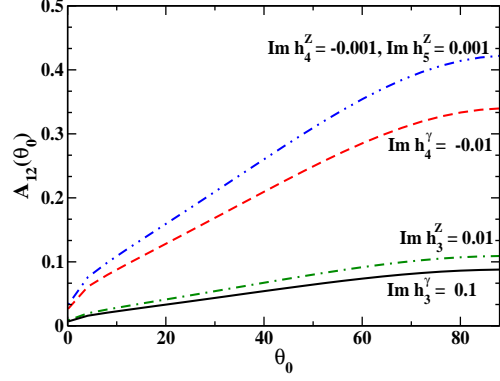


Figure 6. The asymmetry $A_{12}(\theta_0)$ defined in Eq. (4.11) as a function of the cut-off angle θ_0 for the various couplings.

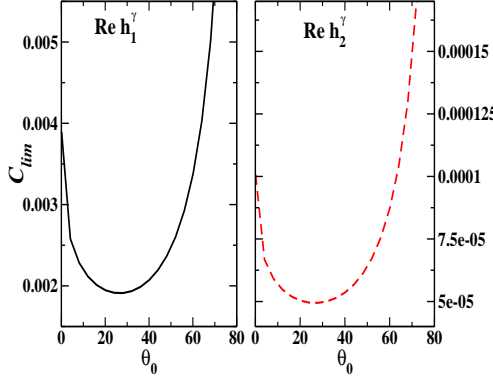


Figure 7. The 90% CL limits on the anomalous couplings from $A_{11}(\theta_0)$, as a function of the cut-off angle θ_0 .

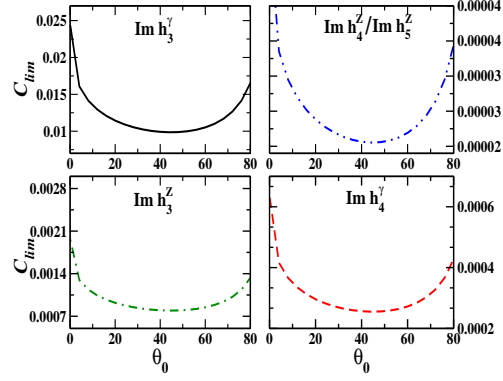


Figure 8. The 90% CL limits on the anomalous couplings from $A_{12}(\theta_0)$, as a function of the cut-off angle θ_0 .

cut-off angle θ_0 . Each coupling is set to a value such that the linear approximation holds good while the other couplings are set to zero. The 90% CL limits obtained on the various couplings from these asymmetries are next shown in Figs. 7 and 8. Similar to the previous case, we see that the limits obtained are independent of θ_0 in the range $20^\circ < \theta_0 < 40^\circ$ in the case of $A_{11}(\theta_0)$. We again consider the optimal value of 30° , with $\text{Re } h_1^\gamma < 2 \times 10^{-3}$ and $\text{Re } h_2^\gamma < 0.5 \times 10^{-4}$. In case of $A_{12}(\theta_0)$, as can be seen from Fig. 8, the limits on $\text{Im } h_{3,4}^\gamma$ and $\text{Im } h_3^Z$ have stable values over a wide range of $20^\circ < \theta_0 < 60^\circ$, with the respective limits being $\text{Im } h_3^\gamma < 1 \times 10^{-2}$, $\text{Im } h_4^\gamma < 2.1 \times 10^{-4}$ and $\text{Im } h_3^Z < 0.9 \times 10^{-3}$. The best limit on $\text{Im } h_{4,5}^Z$ is 2.1×10^{-5} for $\theta_0 = 40^\circ$.

Finally, we present our results for the asymmetries $A_{31,32}(\theta_0)$. The asymmetry $A_{31}(\theta_0)$ as a function of θ_0 , for the various couplings is shown in Fig. 9, with the 90%

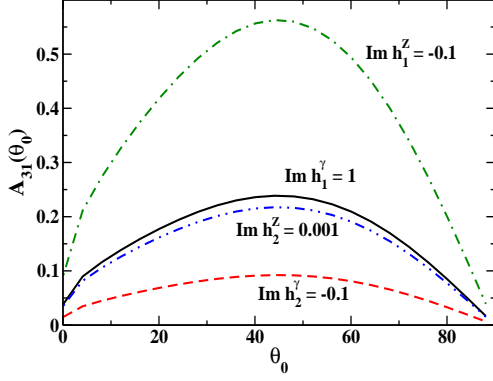


Figure 9. The asymmetry $A_{31}(\theta_0)$ defined in Eq. (4.10) as a function of the cut-off angle θ_0 for the various couplings.

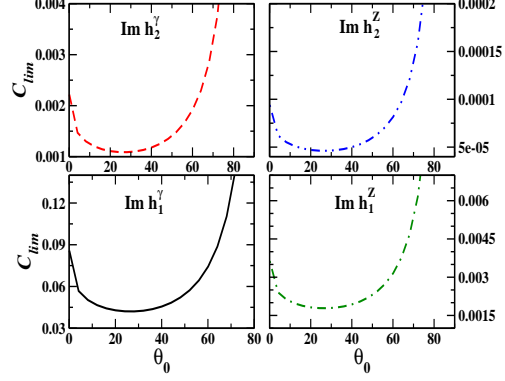


Figure 10. The 90% CL limits on the anomalous couplings from $A_{31}(\theta_0)$, as a function of the cut-off angle θ_0 .

CL limits on the couplings from this asymmetry shown in Fig. 10. The asymmetry $A_{32}(\theta_0)$ contains the SM contribution $A_{32}^{SM}(\theta_0)$ in addition to the contribution from anomalous couplings, so we only plot the contribution from the anomalous couplings defined as $A'_{32}(\theta_0) = |A_{32}(\theta_0) - A_{32}^{SM}(\theta_0)|$. We then determine the individual 90% CL limits on the couplings from $A_{32}(\theta_0)$, using the expression

$$C_{lim} \equiv \frac{1.64 \sqrt{1 - (A_{32}^{SM})^2}}{|A'_{32}| \sqrt{N_{SM}}}, \quad (5.2)$$

where A'_{32} in the denominator is the value of $A'_{32}(\theta_0)$ for unit value of the coupling. The SM contribution to the asymmetry $A_{32}(\theta_0)$ is shown in Fig. 11, and the additional contribution to $A_{32}(\theta_0)$, due to the different couplings apart from the SM, defined as $A'_{32}(\theta_0)$ is shown in Fig. 12. The 90% CL limits obtained on the couplings contributing to $A_{32}(\theta_0)$ from Eq. (5.2) is shown in Fig 13. We only present the result for this case, because the couplings which enter $A_{21,22}(\theta_0)$ are also present in $A_{31,32}(\theta_0)$. It can be seen from Eq. (4.9), that $A_{21}(\theta_0)$ receives contribution from the couplings $\text{Im } h_{1,2}^Z$ and $\text{Im } h_{1,2}^\gamma$, whereas $\text{Re } h_{3,4}^\gamma$ contributes to $A_{22}(\theta_0)$, Eq. (4.12). As these anomalous couplings also contribute to the other asymmetries, and we have checked that the individual limits obtained on these couplings from these asymmetries are of the same order or better than the individual limits obtained from $A_{21,22}(\theta_0)$. Therefore we do not show the results for these asymmetries, but we list in Table 6 the individual limits obtained in this case. Finally we show in Table 7 the best individual limits obtained from the asymmetry $A_{31,32}(\theta_0)$. The limits obtained get better with increase in centre-of-mass energy.

6 Discussion and Conclusions

The gauge sector of the SM is one of the key corners which can provide a window into BSM physics. It has been one that has been studied extensively in the literature. It

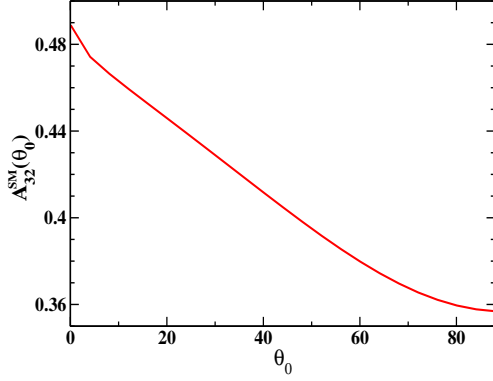


Figure 11. The SM dependent part $A_{32}^{SM}(\theta_0)$ of the asymmetry $A_{32}(\theta_0)$ as a function of the cut-off angle θ_0 .

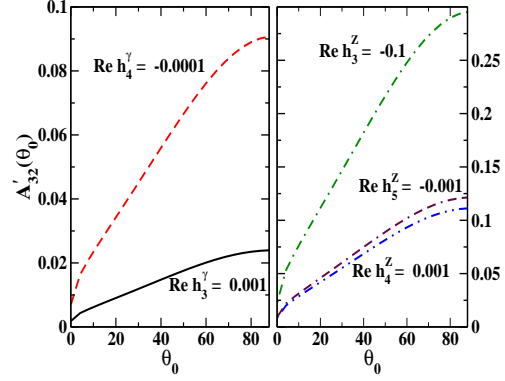


Figure 12. The asymmetry $A'_{32}(\theta_0)$ as a function of the cut-off angle θ_0 for the various couplings.

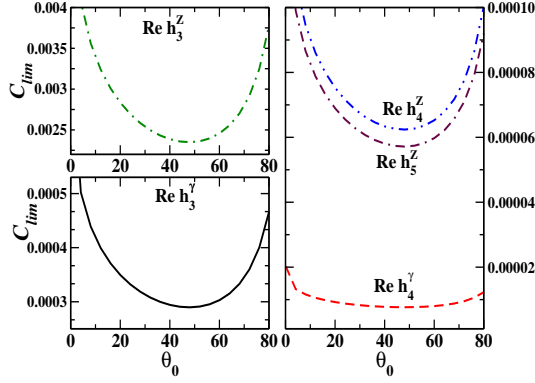


Figure 13. The 90% CL limits on the anomalous couplings from $A'_{32}(\theta_0)$, as a function of the cut-off angle θ_0 .

A_{21}				A_{22}	
$\text{Im } h_1^\gamma$	$\text{Im } h_2^\gamma$	$\text{Im } h_1^Z$	$\text{Im } h_2^Z$	$\text{Re } h_3^\gamma$	$\text{Re } h_4^\gamma$
2×10^{-1}	3×10^{-4}	1.5×10^{-3}	3×10^{-5}	8×10^{-4}	2×10^{-5}

Table 6. Table of sensitivities obtainable at the ILC with the machine and operating parameters given in the text for the asymmetries A_{21} and A_{22} .

has also been probed to high precision at the LEP as well as at the LHC and Fermilab experiments. Anomalous triple gauge boson couplings constitute an interesting and important model-independent method by which BSM physics has been introduced. Another less popular but equally compelling method is to introduce BSM physics through contact interactions. In fact, this latter has not received sufficient attention in the literature.

A_{31}		A_{32}		
$\text{Im } h_1^Z$	$\text{Im } h_2^Z$	$\text{Re } h_3^Z$	$\text{Re } h_4^Z$	$\text{Re } h_5^Z$
1×10^{-3}	2.5×10^{-5}	2×10^{-3}	6×10^{-5}	5×10^{-5}
$\text{Im } h_1^\gamma$	$\text{Im } h_2^\gamma$	$\text{Re } h_3^\gamma$	$\text{Re } h_4^\gamma$	
2×10^{-2}	5×10^{-4}	3×10^{-4}	8×10^{-6}	

Table 7. Table of limits on couplings obtainable at the ILC with the machine and operating parameters given in the text for the asymmetries A_{31} and A_{32} .

One of the missions of the present work is to explore whether anomalous TGCs capture all the essence of BSM physics, or whether one needs to go beyond that. Before embarking on this, we first asked ourselves if the anomalous TGCs considered in the literature are exhaustive or not. It turns out, surprisingly, that from the considerations of Bose symmetry, gauge invariance, etc., it is possible to generate a term that has not been found in the literature. One of the reasons could be that this term is not one that is invariant under the symmetry $Z \leftrightarrow \gamma$. We find a $ZZ\gamma$ coupling, while there is no analogous $Z\gamma\gamma$ term.

While the bounds obtained in [26, 27] might continue to be valid approximately, the analysis of the data clearly would have to be done afresh for more precise bounds in view of the above.

LHC experiments obtain bounds on TGCs by looking at the transverse momentum spectrum of the photon. Since the photon energy spectrum has similar sensitivities to CP-violating and CP-conserving couplings, the LHC cannot discriminate between these couplings. Their results are interpreted in terms of the CP-conserving couplings.

The CP-violating couplings $h_{1,2}^{Z,\gamma}$ can be bounded by studying CP-violating asymmetries, the simplest being the forward-backward asymmetry of the photon in the type of experiments performed at Tevatron. The corresponding effect in $e^+e^- \rightarrow \gamma Z$ was studied in [12]. We have carried out a detailed numerical study of the implications of such BSM physics. We have considered a list of asymmetries, in the presence of both transverse and longitudinal polarization so as to give individual limits on the CP-conserving and the CP-violating couplings. These asymmetries will help to discriminate between the CP-conserving and the CP-violating couplings. Moreover we find that the limits obtained on the TGCs from the various asymmetries will be better than those obtained from the LHC, and will improve with the centre-of-mass energy. In the presence of LP, we find the limits $\text{Im } h_1^{Z,\gamma} < 1 \times 10^{-5}$ and $\text{Im } h_2^{Z,\gamma} < 2.2 \times 10^{-4}$. The limits on the other anomalous couplings are obtained in the presence of TP and are listed in Sec. 5 as well as in Tables 6 and 7.

The two dimension-8 anomalous couplings pertaining to the $ZZ\gamma$ vertex, $h_{4,5}^Z$ show similar behaviour in case of the various asymmetries. At a fixed energy, it

turns out that the distributions are such that the angular behaviour is the same. This is true in the case considered in this work, which is one where the polarization of the two final-state bosons is summed over. It is therefore important to discuss the matter of discriminating between these two anomalous couplings. If it is possible to have an energy scan at the ILC, then the energy dependence would reveal whether the BSM physics is due to h_4^Z or due to h_5^Z .

Alternatively, as in our previous work [17], if the spin of the Z is resolved, it is likely to lead to a situation where one may be able to discriminate between the two sources, since the vertices are actually different. It is clear from Table 1 that the two-index tensor $\Gamma_{\alpha\beta}^Z$ is different for h_4^Z and h_5^Z . Summing over polarizations leads to the same θ and ϕ distributions. Therefore, in order to probe the full tensor structure, it is necessary to resolve the spin(s) of the boson(s). The Z polarization vector, when contracted with the h_4^Z term in Eq. (2.2) or the h_2^γ term in Eq. (2.3), gives a factor $q \cdot \epsilon(k_2)$. In the centre-of-mass frame, q has only the time component, whereas for transverse polarization, $\epsilon(k_2)$ has no time component, the corresponding amplitude vanishes. So only longitudinal polarization for Z contributes to h_4^Z (or h_4^γ). On the other hand, both longitudinal and transverse Z polarizations would survive for the h_5^Z term. It is thus plausible that observing Z polarization can distinguish between h_4^Z and h_5^Z . This is beyond the scope of the present work.

In order to carry out a detailed comparison, we started out by reducing the familiar set of contact interactions to the anomalous TGCs. While the TGCs in the case of CP-conserving interactions were expressed in terms of Levi-Civita terms, and the contact interactions without, we had to carry out a detailed exercise to carry out the comparison. We have established a relation between these two approaches. While doing the analysis we found that a triple gamma term (r_7) which has appeared only once in the literature plays a definitive role. We also found that r_7 has no definite CP transformation property, i.e. the operator multiplying r_7 is partly CP odd and partly CP even. Our conclusions are that anomalous TGC terms do not exhaust all possible distributions that can be generated by contact interactions.

Although our work is motivated by the immediate goal of finding a detailed physics programme for the ILC, it has a more general import. These may be listed as follows:

- (a) A general analysis of the physics of gauge bosons in a model-independent manner, subject only to the constraints of gauge invariance and Lorentz invariance. This is obviously of importance also to the LHC.
- (b) It is of importance to the Compact Linear Collider (CLIC) [35] which also requires a dedicated physics programme, lot of which would be common to the ILC. In the coming years, many of these analyses could be done for CLIC energies and polarization. There would be many distinguishing features between the two as regards the detector capabilities, which are beyond the scope for the present paper.

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A Conversion of anomalous TGCs involving the Levi-Civita symbol

As can be seen from Table 1, some of the anomalous TGC couplings involve the Levi-Civita symbol. The contact interactions discussed in Ref. [28, 29] however do not involve these symbols. Therefore it will be useful to convert the Levi-Civita symbols to a form equivalent to that used for contact interactions only involving the momentum four vectors and the Dirac matrices. We therefore present below a derivation of simplified forms for the anomalous couplings involving $h_{3,4,5}^V$ containing Levi-Civita symbols.

Firstly we would like to observe that the \not{q} terms, occurring singly, can be dropped, because they give 0 on using the Dirac equation for the electron and positron spinors:

$$\bar{v}(p_+)\not{q}u(p_-) = \bar{v}(p_+)(\not{p}_- + \not{p}_+)u(p_-) = 0. \quad (\text{A.1})$$

The terms with $\not{q}\gamma_5$ give zero in the limit of vanishing electron mass, and can also be dropped.

We now take up various terms in Table 1 containing the Levi-Civita tensor by turns. At all stages, we set k_1^α, k_2^β and $q^\nu \equiv (k_1 + k_2)^\nu \equiv (p_+ + p_-)^\nu$ to 0.

1. The $h_3^{\gamma,Z}$ terms have $\Gamma^{\alpha\beta} = \gamma_\nu \epsilon^{\alpha\beta\nu k_1}$. In this term, we can introduce the identity

$$1 = \gamma_5^2 = -\frac{i}{4!} \epsilon^{\rho\lambda\sigma\tau} \gamma_\rho \gamma_\lambda \gamma_\sigma \gamma_\tau \quad (\text{A.2})$$

to get

$$\Gamma^{\alpha\beta} = \frac{i}{4!} \gamma_5 \gamma_\nu \gamma_\rho \gamma_\lambda \gamma_\sigma \gamma_\tau \epsilon^{\alpha\beta\nu k_1} \epsilon^{\rho\lambda\sigma\tau}. \quad (\text{A.3})$$

Of the 5 indices $\nu, \rho, \sigma, \lambda, \tau$ in $d = 4$ at least 2 indices have to be equal. Because of the presence of the totally antisymmetric $\epsilon^{\rho\sigma\lambda\tau}$, only ν will be allowed to be equal to one of $\rho, \sigma, \lambda, \tau$ giving 4 equal terms. Then

$$\Gamma^{\alpha\beta} = \frac{i}{3!} \gamma_5 \gamma_\lambda \gamma_\sigma \gamma_\tau \epsilon_\nu^{\lambda\sigma\tau} \epsilon^{\alpha\beta\nu k_1} \quad (\text{A.4})$$

We now use the identity

$$\begin{aligned} \epsilon_\nu^{\lambda\sigma\tau} \epsilon^{\alpha\beta\nu k_1} &= g^{\lambda\alpha} (g^{\sigma\beta} k_1^\tau - g^{\tau\beta} k_1^\sigma) - g^{\lambda\beta} (g^{\sigma\alpha} k_1^\tau - g^{\tau\alpha} k_1^\sigma) \\ &\quad - k_1^\lambda (g^{\sigma\beta} g^{\alpha\tau} - g^{\tau\beta} g^{\alpha\sigma}) \end{aligned} \quad (\text{A.5})$$

to get

$$\Gamma^{\alpha\beta} = -i\gamma_5(k_1\gamma^\alpha\gamma^\beta + \gamma^\alpha k_1^\beta - k_1 g^{\alpha\beta}) \quad (\text{A.6})$$

2. The $h_4^{\gamma,Z}$ terms have

$$\Gamma^{\alpha\beta} = -\gamma_\nu k_1^\beta \epsilon^{\alpha\nu k_1 q} \quad (\text{A.7})$$

As before using the identity in Eq. A.2, we can write

$$\Gamma^{\alpha\beta} = -\frac{i}{4!}\gamma_5\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\lambda\gamma_\tau\epsilon^{\rho\sigma\lambda\tau}k_1^\beta\epsilon^{\alpha\nu k_1 q}, \quad (\text{A.8})$$

which on using the fact that one pair of 5 indices has to be equal, gives

$$\Gamma^{\alpha\beta} = -\frac{i}{3!}\gamma_5\epsilon_\nu^{\sigma\lambda\tau}\gamma_\sigma\gamma_\lambda\gamma_\tau k_1^\beta\epsilon^{\alpha\nu k_1 q} \quad (\text{A.9})$$

This equation reduces, on using the identity for a product of two Levi-Civita tensors to be written in terms of Kronecker deltas, to

$$\Gamma^{\alpha\beta} = -\frac{i}{3!2}\gamma_5 k_1^\beta (\gamma^\alpha k_1 \not{q} - \gamma^\alpha \not{q} k_1 - k_1 \gamma^\alpha \not{q} + k_1 \not{q} \gamma^\alpha + \not{q} \gamma^\alpha k_1 - \not{q} k_1 \gamma^\alpha) \quad (\text{A.10})$$

We now use $\not{q} = \not{p}_- + \not{p}_+$, then anti-commute \not{p}_- to the extreme right and \not{p}_+ to the extreme left, and use $\not{p}_- u(p_-) = 0$; $\bar{v}(p_+) \not{p}_+ = 0$. Dropping k_1^α terms because ($\epsilon_\alpha(k_1)k_1^\alpha = 0$), the result is

$$\Gamma^{\alpha\beta} = -i\gamma_5 k_1^\beta (p_+ \cdot k_1 \gamma^\alpha - p_- \cdot k_1 \gamma^\alpha - p_+^\alpha k_1 + p_-^\alpha k_1) \quad (\text{A.11})$$

3. The h_5^Z term is

$$\Gamma^{\alpha\beta} = \gamma^\nu 2k_2^\alpha \epsilon^{\beta k_2 \nu q} - \frac{(s - m_Z^2)}{2} \gamma^\nu (\epsilon^{\alpha\beta k_2 \nu} + \epsilon^{\alpha\beta q \nu}) \quad (\text{A.12})$$

The first term of Eq. A.12, following the same procedure as before yields

$$2\gamma_\nu k_2^\alpha \epsilon^{\beta k_2 \nu q} = -\frac{4i}{3!} k_2^\alpha \gamma_5 [\gamma_\beta (\not{k}_2 \not{q} - \not{q} \not{k}_2) - \not{k}_2 \gamma_\beta \not{q} + \not{q} \gamma_\beta \not{k}_2 - \not{q} \not{k}_2 \gamma_\beta + \not{k}_2 \not{q} \gamma_\beta]. \quad (\text{A.13})$$

Now using $\not{q} = \not{p}_- + \not{p}_+$ and then commuting \not{p}_+ through to the extreme left and \not{p}_- to the extreme right and using the Dirac equation, the right-hand side of Eq. A.13 becomes

$$2i\gamma_5 [(p_- \cdot k_2 - p_+ \cdot k_2) \gamma_\beta - (p_- - p_+)_\beta \not{k}_2] k_2^\alpha \quad (\text{A.14})$$

The second term of Eq. A.12 simplifies to

$$\gamma^\nu \epsilon_\nu^{\alpha\beta k_2} = i\gamma_5 [\gamma^\alpha \gamma^\beta \not{k}_2 - g^{\alpha\beta} \not{k}_2 + k_2^\alpha \gamma^\beta] \quad (\text{A.15})$$

Similarly the third term of Eq. A.12 is

$$\gamma_\nu \epsilon^{\alpha\beta q \nu} = i\gamma_5 [(p_+ - p_-)^\beta \gamma^\alpha + (p_- - p_+)^\alpha \gamma^\beta] \quad (\text{A.16})$$

Combining Eqs. A.14, A.15 and A.16, $V_{\alpha\beta}$ for the h_5^Z term takes the form

$$\begin{aligned} & i\gamma_5 [2k_2^\alpha (p_- - p_+) \cdot k_2 \gamma^\beta - 2k_2^\alpha (p_- - p_+)^\beta \not{k}_2 \\ & + \frac{(s - m_Z^2)}{2} \{(\gamma^\alpha \gamma^\beta \not{k}_2 - g^{\alpha\beta} \not{k}_2 + k_2^\alpha \gamma^\beta) \\ & + ((p_- - p_+)^\alpha \gamma^\beta - (p_- - p_+)^\beta \gamma^\alpha)\}] \end{aligned} \quad (\text{A.17})$$

In Eq. A.17, \not{k}_2 can be replaced by \not{k}_1 using $\not{k}_2 = \not{q} - \not{k}_1$. Then, using, as before, $\not{q} = \not{p}_- + \not{p}_+$, and the Dirac equation after appropriately commuting through \not{p}_- and \not{p}_+ to the extreme right and left, respectively, we get

$$\begin{aligned} \Gamma^{\alpha\beta} = & i\gamma_5 [2k_2^\alpha \{-k_1 \cdot (p_- - p_+) \gamma^\beta + (p_- - p_+)^\beta \not{k}_1\} \\ & + \frac{(s - m_Z^2)}{2} \{-\not{k}_1 \gamma^\alpha \gamma^\beta + k_1^\beta \gamma^\alpha - +2(p_- - p_+)^\alpha \gamma^\beta g^{\alpha\beta} \not{k}_1 - 2(p_- - p_+)^\beta \gamma^\alpha\}] \end{aligned} \quad (\text{A.18})$$

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